Nigerian Journal of Environmental Sciences and Technology
(NIJEST)
www.nijest.com
ISSN (Print): 2734-259X | ISSN (electronic): 2734-2603
Vol 7, No. 1 March 2023, pp 167 - 175

Error Budget Analysis in Gravity Network Adjustment

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https://doi.org/10.36263/nijest.2023.01.0407

ABSTRACT

The Minimum quadratic unbiased estimate (MINQUE) Variance Component Estimation (VCE) technique has not been used in gravity network adjustment and is therefore herein proposed for computing the error budget in the adjustment of a regional gravimetric network using numerical examples from terrestrial gravity data covering the south western region of Nigeria. Suitable parametric model was chosen to enhance analysis of the gravity dependent, position dependent and height dependent error contributions in the resulting gravity network. Results obtained show variance components of $\pm 0.001\ mgals, \pm 2.222mm,\ \pm 0.554mm$ *and* $\pm 0.296m$ *in the gravity, Latitude, longitude and height data types. It is concluded that while low accuracy horizontal positioning may be acceptable, centimeter level accuracy is required for heights if 0.001mgals accuracy is to be achieved in regional gravity network.*

Keywords: Variance Component Estimation (VCE), MINQUE, gravity network, Ordinary Least Squares (OLS)

1.0. Introduction

Terrestrial gravity observations taken at spatially varied points across the earth surface are important quantities used by several disciplines especially in geosciences and geotechnical engineering (Gooma, 1998; Odumosu et al, 2013). Gravity surveys (just as other geodetic exercises) requires that field measurements are properly tied to a control network known as "Gravity standardization Network" whose parameters (gravity, scale, orientation and calibration values within the reference frame) are precisely known (Torge, 2001); after which such gravity controls can be densified or extended to other areas.

Relative gravity measurements are fundamental observations for determining gravity values. It is common to collect redundant observations to ensure a better quality of gravity determination, and hence gravity measurements need to be adjusted using adequate mathematical and stochastic models (Hwang et al, 2002).The establishment of a regional gravity network depends basically on development of appropriate mathematical and stochastic models based on network configuration/geometry for preliminary and post processing estimation, assessment of optimal method for estimation and assessment of quality mathematical models and optimization criteria (Peneva et al, 2015).

Numerous mathematical models have been proposed by various researchers to detect gross, random and systematic errors in gravity networks e.g., Baarda (1966), Koch (1987), Caspary (2000), and Dudewicz and Mishra (1988). However, the choice of suitable stochastic models for network error assessment remains an area of research that has not been satisfactorily solved. This will therefore be the subject of discussion in this paper.

In this work, the well-studied variance component estimation (VCE) technique is used as a method for calibrating gravimetric error model in a gravity network. Implementation of VCE for the adjustment of heterogeneous gravity datasets has not yet been addressed in geodetic literature. The VCE technique is herein suggested because it will facilitate easy assessment of the noise derived i.e. testing the noise level and allow determination of the error budget from each dataset used in the adjustment. Guo and Xu (2015) used the VCE approach in a combined LS adjustment with heterogeneous height data but in this work, similar procedure is used to conduct an error budget analysis for gravity network adjustment.

The VCE is an effective statistical tool that is flexible and easy to use (Fotopoulos, 2003). Since network adjustments begin with the mathematical model (considering the kind of data to be adjusted in this work i.e. point gravity data from various sources), the Gauss-Markov functional model shall be used in this study along with the Minimum Quadratic Unbiased Estimate (MINQUE) estimation procedure.

2.0. Methodology

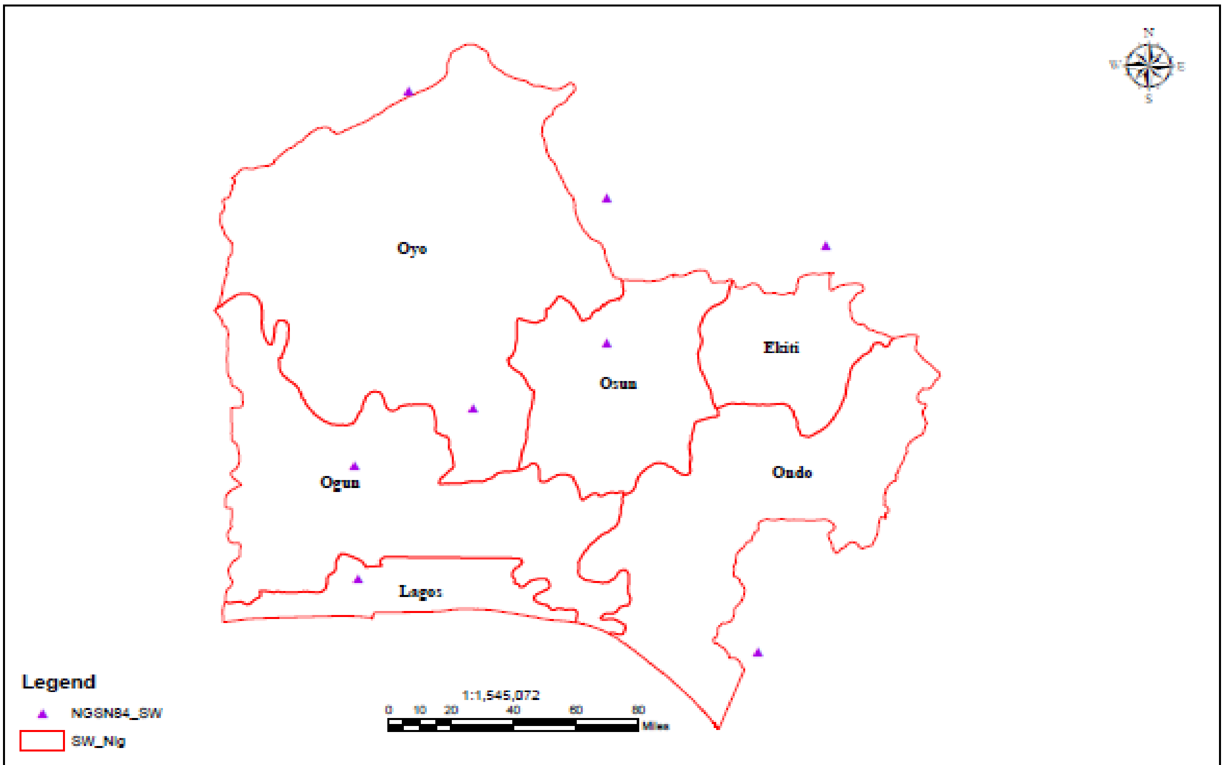
2.1. Study Area

The study area is the southwestern geopolitical region of Nigeria (Figure 2). The South west geopolitical zone consists of six (6) states namely: Lagos, Ogun, Oyo, Osun, Ekiti and Ondo states each having a land mass of 3577 sq km, 16,980.55 sq km, 28,454 sq km, 9,251 sq km, 6,353 sq km and 15,500 sq km respectively. There are 7 gravity control stations (part of the NGSN-84) located within the study area. Unfortunately, due to several human activities, most of these points have been lost except 103901 which was also part of the IGSN-71 points.

Table 1: NGSN-84 stations within the study area.

STATION NUMBER	NAME	LAT	LONG	ABS GRAV (mGal)	S.D (mGal)
100101	ABEOKUTA	7.133	3.317	987119.9	0.021
100501	AKURE	6.233	5.183	978050.3	0.018
102301	IBADAN	7.417	3.867	978089.2	0.018
102701	ILORIN	8.433	4.483	978059.1	0.017
102801	ISEYIN	8.950	3.567	978076.5	0.020
103901	LAGOS	6.583	3.333	978114.4	0.016
105001	OSOGBO	7.733	4.483	978063.9	0.016

Adapted from Osazuwa, 1995



Spatial distribution of gravity observation stations across SW Nigeria

Figure 1: Study area and the IGSN stations.

2.2. Data description

Two different datasets were used for this exercise. The aim of this network readjustment exercise was to homogenize the different gravity datasets within the study area with a view to preparing a recent and scientifically consistent regional geoid. A total of 1256 terrestrial gravity data covering the study area was collected from various sources and used in this study. Crossover adjustments and loop closure analysis (data pre-adjustment) was done to remove outliers and points that are not consistent with the NGSN-84 points (Odumosu et al, 2015) leaving 191gravity stations as homogenous points. The data used are as tabulated in table 2.

Table 2: Data used

S/N	Data Source	Number of points	Accuracy (mgals)	Date of Observation
1	Geological survey of Nigeria (BGI)	987	Not available	1/11/1961
2	Shell Exploration Company (BGI)	69	Not available	1/1/1965
3	University of Ibadan (BGI)	192	Not available	1/1/1978
4	NGSN-84 (Osazuwa, 1995a)	8	0.015	1979 - 1983

2.3. Methods

2.3.1 Parametric model and associated VCE formulations

To establish the LS-VCE technique for gravimetric network data adjustment, the conventional Gauss Marckov functional model has been used in this paper. The mathematical combination adopted comprises of the contributions of the errors in gravity scale within the network, wrong height value (orthometric height) and incorrect position determination.

Given a homogenous gravity dataset scattered across different points on the earth surface, along with the heights and positions at such points, a unified adjustment of such data distribution into a single constrained gravity network will require that the Gauss-Markov functional model be used wherein the system of observation equations and the solution can be expressed as:

$$AX + V = L^a \quad (1a)$$

$$E\{v\} = 0 \quad (1b)$$

$$E\{vv^T\} = C_v = \sigma^2 Q_v \quad (1c)$$

Where:

A = $m \times t$ matrix (depending on the parametric model) of known coefficients known as the design Matrix.

X = t × 1 vector of unknown parameters

V = vector of residuals

m = Number of observation equations

t = Number of unknown parameters

$E\{.\}$ = is the mathematical expectation operator

C_v = covariance matrix

σ^2= variance components

Q_v= Cofactor matrix of observables

Considering the network configuration, the parametric model used in this research is:

$$\Delta g = x_0 + x_1\theta + x_2\lambda + x_3 h \quad (2)$$

Where:

Δg = gravity anomaly

θ = latitude

λ = longitude

h = Orthometric height

Therefore, equation (2) was used in building the design matrix

The vector of observable random errors (v) for each of the heterogeneous data types (gravity, height and position) is given by equation (3):

$$v = v_g + v_\theta + v_\lambda + v_h \qquad (3)$$

Therefore from (1c) the corresponding covariance matrix (C_v) can be described by equation (4):

$$C_v = \begin{bmatrix} C_g & 0 & 0 & 0 \\ 0 & C_\theta & 0 & 0 \\ 0 & 0 & C_\lambda & 0 \\ 0 & 0 & 0 & C_h \end{bmatrix} = \begin{bmatrix} {\sigma_g}^2 Q_g & 0 & 0 & 0 \\ 0 & {\sigma_\theta}^2 Q_\theta & 0 & 0 \\ 0 & 0 & {\sigma_\lambda}^2 Q_\lambda & 0 \\ 0 & 0 & 0 & {\sigma_\lambda}^2 Q_h \end{bmatrix} \qquad (4)$$

It should be noted that the obtained CV matrix could be implemented in a least squares collocation (LSC) to determine the vector of unknown parameters using the conventional LSC formulae given in equation 5:

$$X = (A^T \bar{C}^{-1} A)^{-1} A^T \bar{C}^{-1} L \qquad (5)$$

Where:

$\bar{C}^{-1}$ = inverse of the CV Matrix in (4)

L = vector of observations

The mean square error of parameters is then given by (6)

$$\widehat{\sigma_0} = \sqrt{\frac{{v_g}^T {C_g}^{-1} v_g + {v_\theta}^T {C_\theta}^{-1} v_\theta + {v_\lambda}^T {C_\lambda}^{-1} v_\lambda + {v_h}^T {C_h}^{-1} v_h + {v_h}^T {C_h}^{-1} v_h}{m-t}} \qquad (6)$$

2.3.2 The MINQUE Implementation Procedure

The minimum norm quadratic unbiased Estimation procedure is one of the many VCE estimation procedures (Rao, 1971; Sjoberg, 1983) but is preferred in this work since it does not require any form of distributional assumption (Guo and Xu, 2015). Generally, the MINQUE algorithm begins with the ordinary least squares (OLS) using the chosen parametric model to determine initial estimates that are iteratively solved in the VCE process.

The general MINQUE equation is given in equation (7):

$$S\theta = q \qquad (7)$$

Where:

θ = vector of unknown variance components ($\sigma_g, \sigma_\theta, \sigma_\lambda, \sigma_h$)

$$S = \begin{pmatrix} S_{gg} & S_{g\theta} & S_{g\lambda} & S_{gh} \\ S_{\theta g} & S_{\theta\theta} & S_{\theta\lambda} & S_{\theta h} \\ S_{\lambda g} & S_{\lambda\theta} & S_{\lambda\lambda} & S_{\lambda h} \\ S_{hg} & S_{h\theta} & S_{h\lambda} & S_{hh} \end{pmatrix}$$

$$S_{ij} = tr(RQ_iQ_j)$$

Q = cofactor matrix of observables

$$R = C_1^{-1}[I - A(A^T C_1^{-1} A)^{-1} A^T C_1^{-1}]$$

A = design Matrix

C_1 = Initial estimate of the CV matrix

Therefore equation (7) is implemented explicitly as

$$\begin{pmatrix} tr(RQ_gRQ_g) & tr(RQ_gRQ_\theta) & tr(RQ_gRQ_\lambda) & tr(RQ_gRQ_h) \\ tr(RQ_\theta RQ_g) & tr(RQ_\theta RQ_\theta) & tr(RQ_\theta RQ_\lambda) & tr(RQ_\theta RQ_h) \\ tr(RQ_\lambda RQ_g) & tr(RQ_\lambda RQ_\theta) & tr(RQ_\lambda RQ_\lambda) & tr(RQ_\lambda RQ_h) \\ tr(RQ_hRQ_g) & tr(RQ_hRQ_\theta) & tr(RQ_hRQ_\lambda) & tr(RQ_hRQ_h) \end{pmatrix} \cdot \begin{bmatrix} \hat{\sigma}^2{}_g \\ \hat{\sigma}^2{}_\theta \\ \hat{\sigma}^2{}_\lambda \\ \hat{\sigma}^2{}_h \end{bmatrix} = \begin{bmatrix} w^T R^T Q_g{}^T Rw \\ w^T R^T Q_\theta{}^T Rw \\ w^T R^T Q_\lambda{}^T Rw \\ w^T R^T Q_h{}^T Rw \end{bmatrix}$$

Where w = imposed weights

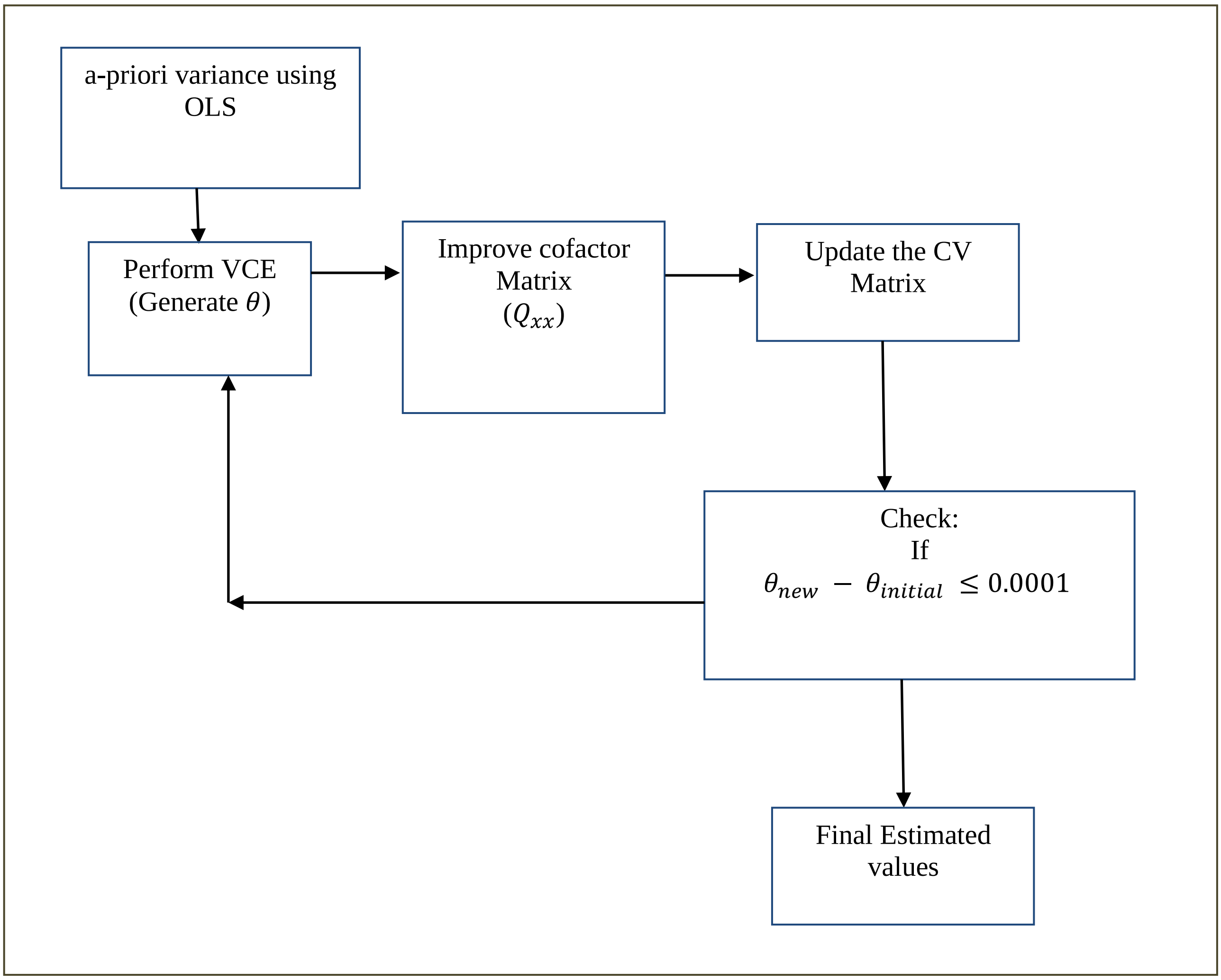


Figure 2: Iterative procedure for MINQUE VCE (modified after Guo and Xu, 2015)

3.0. Results and Discussion

Presented in Table 3 is a statistics of the FA gravity anomaly before and after data pre- adjustment while table 4 shows the obtained CV matrix characteristics for the network of gravity points after the VCE converged. Table 5 gives the initial characteristics of the gravity network as obtained from the initial ordinary least squares (OLS).

Table 3: Statistics of FA anomaly before and after pre-adjustment of data

	Min	Max	Mean	Range	No_Points
Before Pre-adjustment	-32.5	73	8.74	105.5	1256
After Pre-adjustment	-25.5	56	27.81	81.5	191

Table 4: Characteristics of CV matrix after the convergence of the VCE procedure.

Parameter	Value
Gravity (σ_g)	0.001 mgals
Latitude (σ_θ)	0.0002 deg =2mm
Longitude (σ_λ)	0.00005 deg = 0.5mm
Height (σ_h)	0.296m

Table 5: Characteristics of initial OLS

Parameter	Value
Standard deviation (σ^2)	$\pm$0.768mgals
Sum of residuals ($\sum v$)	-107.698
Sum of squares of residuals ($\sum v^2$)	20190.925

Analysis of tables 4 and 5, we see that the contribution of the horizontal positional error on the cumulative standard deviation of the network is largest, followed by the height dependent error. This shows that gravity network adjustment is not entirely independent of the position determination accuracy and further suggests that for an efficient regional gravity network, accurately observed position and height data is a prerequisite. Therefore, since no information exists as to the accuracy of position determination, it is assumed that the error contribution from the positional data is more than that from the gravity observations.

The results further suggests that for reliable gravity network surveys ($\leq \pm 0.001$ mgals), mean horizontal position accuracy of the points of the network should not exceed $\pm\ 14m$ while vertical positional accuracy should not exceed $\pm$0.3m . This justifies the use of handheld GNSS receivers and inbuilt GNSS receivers of the scrintrex CG5 for horizontal positioning during gravity surveys (Scrintrex Manual, 2005). Extra care is however expected during leveling (heighting operation) as a greater magnitude of data accuracy is required in heights for gravity surveys. This is due to the close dependence of gravity reductions and subsequent computations on height.

4.0. Conclusions

Since the main idea of this paper is to discuss the error budget analysis in gravimetric network adjustment, it can be concluded that the use of the iterative MINQUE VCE procedure is optimal in detecting the data type with greatest contributing error in gravity network adjustment. The numerical tests from this work show variance components of $\pm 0.001\ mgals, \pm 2.222m,$ $\pm 0.554m$ and $\pm 0.296m$ in the gravity, latitude, longitude and height data types.

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Cite this article as:

Abiodun, O.A., Eghiator-Irughe, R., Osuji S. and Odumosu, J.O., (2023). Error Budget Analysis in Gravity Network Adjustment. *Nigerian Journal of Environmental Sciences and Technology*, 7(1), pp. 167-175. https://doi.org/10.36263/nijest.2023.02.0407