



Comparison of Three Selected Interpolation Models for Hybrid Geoid Determination in Rivers State, Nigeria.

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<https://doi.org/10.36263/nijest.2023.1.0415>

ABSTRACT

The main thrust of this research is to investigate comparatively three (3) selected interpolation techniques for hybrid geoid modeling in Rivers State, Nigeria. The approach deployed is hinged on the Ex-Post factor research design demonstrated in the hybrid geoid model, which involves the integration of geometric geoid and gravimetric geoid models obtained from the zonal harmonics of the earth gravity field (EGM 2008), using a surface interpolation technique as independent variables. The results of the study gave the root mean squares error of 0.00035, 0.000645 and 0.000894 for the inverse distance weighting interpolation, kriging interpolation method and Radial Basis Function respectively. The 3-D surface model of the three interpolation models were generated. The result analysis reveals that the Kriging Interpolation model is the most optimal interpolation models for hybrid geoid modeling in Rivers State.

Keywords: Hybrid, Interpolation, Geoid, Kriging.

1.0. Introduction

The three-dimensional position of a point is cardinal for any engineering project, deformation monitoring, property boundary demarcation, navigation, and supply chain management (Hart, Basil, and Oba, 2021). It forms the bedrock for any meaningful and sustainable developmental project (Fubara, 2011). The three-dimensional position expressed in terms of geodetic latitude, longitude, and ellipsoidal height of points can be determine readily by any user fitted with a GNSS receiver (Kaplan and Hegarty, 2017). However, the height obtained from GNSS (ellipsoidal height) are purely geometrical quantities that have no connection to earth gravity potentials; and it is gravity potential that determines how water flows (Ikharo, Ono and Saheed, 2019). By implication, two points of same ellipsoidal height may be at different equipotential level. Hence ellipsoidal height can't be used for design and construction of engineering facilities. It is indeed a theoretical height reckon from a reference ellipsoid. As such, there are many different types of ellipsoidal heights as there are different types of ellipsoids. The height of point defined by the direction of earth gravity field vector is the orthometric height. The orthometric height are primarily determined from the combination of spirit levelling and gravity measurement (Moka, 2011). Spirit levelling is no doubt one of the most accurate methods of surveying. However, it is very tedious and time consuming and requires an adequate and even distribution of benchmark. Hence, in recent time, there have been a strong advocacy for orthometric height determination from GNSS surveying using a good geoidal model (Opaluwa and Adejare, 2010; Herbert and Olatunji, 2021).

However, the accuracy of the so derived orthometric height from GNSS surveying is predicated on the quality of the geoidal model (Lee et al., 2021), and the interpolation method used (Arana, et al., 2017). The aim of this study is to demonstrate the applicability of selected interpolation techniques in the hybrid geoid model determination in Rivers State. The specific objectives hereto are to develop a hybrid geoid model from the combination of geometric and gravimetric geoidal heights of 56 common points and the integration of same using selected interpolation techniques and their associated reliability. In addition, the graphical representation of the model surface with respect to the interpolation techniques will be obtained. The hybrid geoid model involves the integration of geometric geoid and zonal harmonic coefficient of the earth gravity field (EGM 2008), using a surface interpolation technique (Arana et al., 2017).

1.1 Study Area

The study area lies between Latitude 04° 15'N to 04° 25'N and Longitude 05° 20'E and 07° 15'E. It covers about eight local government area which include Port Harcourt, Oyigbo, Okrika, Ogu-bolo, Obio-Akpor, Ikwerre, Etche, and Eleme of Rivers State, Known as the greater Port Harcourt (Basil, 2021).

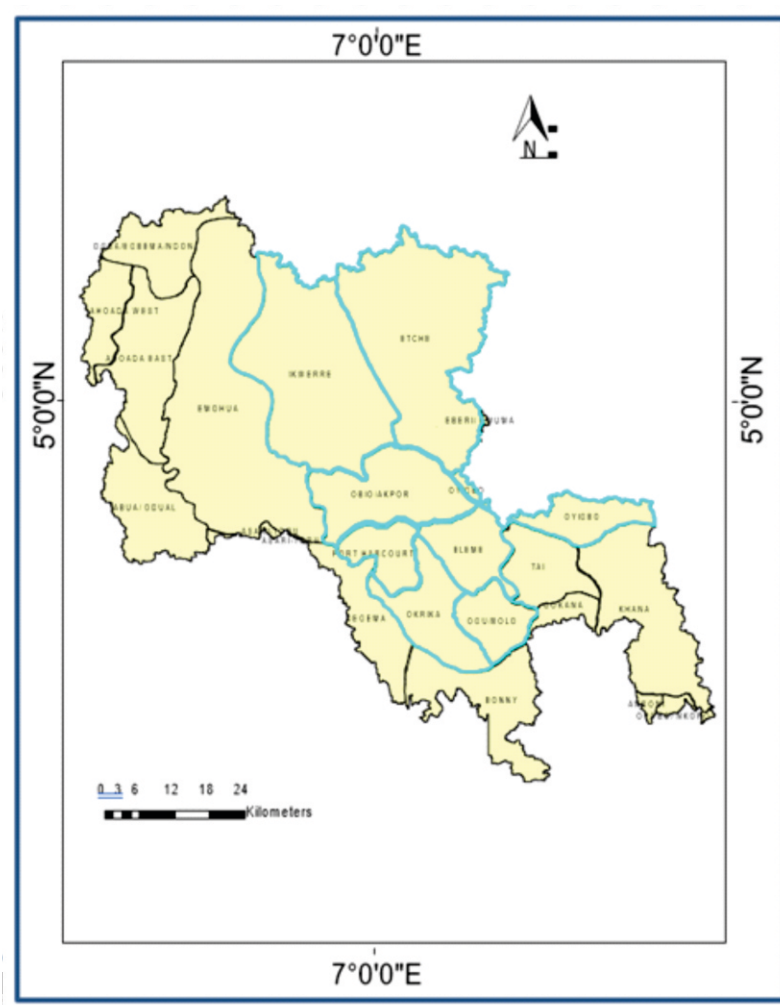


Figure 1: the study area.

2.0 Methodology

The integration of the geometric model and the gravimetric model is called hybridization and the model form is the hybrid model (Arana et al., 2017; Pa'suya, et al., 2022). The input data for the hybrid model generation are GPS/Levelling geoid height ($N_{\text{geometric}}$) and a gravimetric geoid grid (N_{EGM2008}) obtained from the spherical harmonic coefficient of the earth gravity field. The hybridization is a process in three steps, which include the computation of the offset between the geometric geoid undulation derived from the GNSS/Levelling and that computed from the zonal harmonic coefficient (gravimetric geoid undulation) of the earth gravity field on the common point. secondly, the creation of a correctional surface using the obtained difference between the geometric and the gravity geoid as expressed in equation (2); and lastly, the correctional surface is fitted with the gravimetric geoid grid (Arana et al., 2017; Pa'suya, et al., 2022).

The offsets calculated according to Equation (2) are regarded as differences (ΔN) to the gravimetric geoid model. The geoidal undulation implied by EGM 2008 ($N_{\text{EGM 2008}}$) can be obtained as given in equation (1) by (Pavlis, Holmes, Kenyon, and Factor, 2008);

$$N_{\text{EGM}} = \frac{GM}{r\gamma} \sum_{n=2}^{n_{\text{max}}} \left(\frac{a_{\text{ref}}}{r} \right)^n \sum_{m=0}^n (\ddot{C}_{nm} \cos m\lambda + S_{nm} \sin m\lambda) \ddot{P}_{nm}(\cos \vartheta) \quad (1)$$

$$\Delta N = N_{\text{geometric}}(\vartheta, \lambda) - N_{\text{EGM2008}}(\vartheta, \lambda) \quad (2)$$

Where, GM is the product of the mass constant of the earth, r is the geocentric distance of the computation points from the earth center, a_{ref} is the associated scaling parameter for the particular GGM (EGM 2008), $P_{nm}(\cos \vartheta)$ are fully normalized Legendre functions of degree n and order m, \ddot{C}_{nm} and \ddot{S}_{nm} are fully normalized zonal harmonic of the reference ellipsoid and n_{max} is the finite maximum degree of GGM (EGM 2008).

However, to combine the difference obtained with the derived geoid height from the geopotential model we need to generate a regular grid surface in other to combine them. This combine surface would serve as the base for which any interpolation for any point would be determined as shown in Equation (3):

$$N_{\text{Hybrid}} = N_{\text{EGM2008}}(\vartheta, \lambda) + \Delta N(\vartheta, \lambda) \quad (3)$$

The refinement of the developed hybrid model is predicated on the correctional surface so developed which is hinged on the surface representation (interpolation methods). This underscores the critical of the choice of an interpolation model in our quest to model the hybrid geoid of Rivers State, Nigeria. The research method was viewed within the frame of the ex-post facto design as the independent variables (gravimetric and geometrical approaches) in this research were situated to assess their impact on the dependent variable (hybrid). This is also applicable on the methods of interpolation techniques deployed to underscore the recoverability of the original values of the geoid model.

2.1 Interpolation Methods

Interpolation is the determination or the estimation of the values of a given phenomenon at

location in which we have no information about a set of sample data (Hart, 2015). For this study, the various interpolation algorithms which include the inverse distance weighting, radial base function, kriging, nearest neighborhood, minimum surface curvature, modify speared method is investigated to determine statistically the most optimal method for the determination of a hybrid geoid for Rivers State.

2.2 Inverse Distance Weighting (IDW) Interpolation Method

Inverse distance weighting (IDW) is a type of deterministic method; that is, it creates a continuous surface by only using the geometric characteristics of point observations for multivariate interpolation with a known scattered set of points. IDW rely on Toblers first law of geography which posit that everything is related to all else, and that closer to each other are even more related (Tobler, 1970). The application of Tobler's law is even more in the field of spatial science and geostatic where it is required to model the spatial characteristic of a region of interest from observation made within the area. Inverse distance weight computed the spatial characteristic of an unknown point using the weighted distance from known point. Hence the computed value of the unknown point is influence by the closest point to the unknown points as the weight matrix is inversely proportional to the distance from the known point (Lam, 1983, Hart, 2015; Ojigi, 2011). The Equation for the interpolation with the inverse distance weighting is given as in Equation (4) by Davis (1986):

$$\hat{Z}_j = \frac{\sum_{i=1}^n \frac{Z_i}{d_{ij}^\beta}}{\sum_{i=1}^n \frac{1}{d_{ij}^\beta}} \quad (4)$$

Where, d_{ij} is the effective distance between the grid node “j” (the interpolated point) and the neighboring point “i” (Known points); Z_j is the interpolated values for the grid node “j” β is the weighting power (the power parameter). Most commonly, the function used is the inverse squared of distances.

2.3 Kriging Interpolation Method

Kriging Interpolation techniques is among the of group statistical modeling in geostatic that has enjoyed wide application for modeling spatial characteristics of unknown points using the kriging weight computed for the known point (Ogundere, 2019;). Kriging is not an exact interpolation method in that it does not produce an interpolating surface that does honor the observations at the station locations (Nynke, et al., 2008). Kriging interpolation algorithm involves two stages; the determination of the spatial covariance matrix called the variogram. Secondly, the weighting derived from the spatial covariance matrix is used to compute the interpolated value of the unknown points. A variogram (sometime called a semi-variogram) is a visual depiction of the covariance exhibited between each pair of points in the sampled data, which can either be a linear semi-variogram, spherical semi-variogram, exponential semi-variogram or the power semi-variogram. The choice of which is fundamentally user defined. The kriging weight for each interpolated point are calculated from the semivariogram based on the spatial structure of the data. The semivariogram is obtained using the Lagrangian minimization to obtain the best linear unbiased estimate (BLUE) as given in equation (5), which is used in the computation of the sample point as expressed in equation (6) (Deutsch and Journal 1992; Ogundere, 2018).

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z(x_i) - z(x_i + h)]^2 \quad (5)$$

$$\hat{z}(x_i) = \sum_{j=1}^n \lambda_j z(x_j) \quad (6)$$

Where: $\gamma(h)$ is the semi-variogram, h is the lag distance or step length. $N(h)$ is the number of sample points whose distances is equal to h . $z(x_i)$ and $z(x_i + h)$ the respectively regionalized variable, $\hat{z}(x_i)$ is the interpolated value, λ_j is the kriging weight determined from the semi-variogram and $z(x_j)$ is the values of known sample points within the spatial field.

2.3 Radial Basis Function

Radial basis Function (RBF) interpolation is a diverse method of data interpolation (Ojigi, 2011) is based on some mathematical models inspired by the neural structure of intelligent organism identifying patterns recognition, rating, and interpolation (Wang, 2003) The functions use distances as input and from this variable a number of neurons are activated (Wright 2003). The interesting attribute of the radial basis function is that they are exact interpolator that create an interpolation surface that honor the observations at the station locations. Equation (7-11) gives the mathematical model of some types of RBF (Ojigi, 2011; Arana, et al., 2017):

$$\text{Inverse Multiquadric } B(h) = \frac{1}{\sqrt{(h^2 + R^2)}} \quad (7)$$

$$\text{Multilog } B(h) = \log(h^2 + R^2) \quad (8)$$

$$\text{Multiquadric } B(h) = \sqrt{(h^2 + R^2)} \quad (9)$$

$$\text{Natural Cubic Spline } B(h) = (h^2 + R^2)^{3/2} \quad (10)$$

$$\text{Thin Plate Spline } B(h) = (h^2 + R^2) \log(h^2 + R^2) \quad (11)$$

Where h is the anisotropically rescaled, relative distance from the point to the node, R^2 smoothing factor specified by the user. The default value for R^2 in the radial basis function gridding algorithm is calculated as given by (Powell, 1990):

$$R^2 = \frac{L \times 2}{25 \times n} \quad (12)$$

Where L is the length of the diagonal of the data extent, and n is the number of data point.

2.5 Statistical Test

The Mean Absolute Error (MAR), the Root Mean Deviation (RMS), the Mean Relative Error and the coefficient of correlation were used as the statistical indicator to compare the various interpolation model in this research. The MAR is an unambiguous indication of the average error of the estimate (interpolated) value, it is given as shown in Equation 13 by (Willmott and Matsuura, 2006):

$$MAE = \frac{1}{n} \sum_{i=1}^n ABS(Z_{ai} - Z_{ei}) \quad (13)$$

The Root Mean Squared Deviation (RMSD) also known as the root mean squared error (RMSE) is a commonly used measure of the deviation of a predicted value from an observed value. The root mean deviation is a measure of accuracy of the predicted values as given in Equation 14 (Ogundere, 2019; Ghilani and Wolf, 2006):

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (Z_{ai} - Z_{ei})^2}{n-1}} \quad (14)$$

The relative mean error (MRE) is computed to help judge the goodness of fit. The MRE reflects the accuracy of the estimated values against the observed values. Equation 15 gives the mathematical model for the computation MRE as given by (Liu and Yan, 2021):

$$MRE = \frac{1}{n} \sum_{i=1}^n \frac{ABS(Z_{ai} - Z_{ei})}{Z_{ei}} \quad (15)$$

3.0 Presentation and Discussion of Results

The gravimetric geoid undulation was computed according to equation (1) via Geoid Eval Software and UNAVCO online software. The input was the geodetic coordinate, and the output were the geodetic coordinates with their corresponding geoidal height. The result of the computation is given in table 4.1. This is in line with objective one of this research.

Table 1: Showing a Sample of the Station Points and their corresponding EGM 2008 Geoid Height

Stations	LAT. (φ) Decimal degree	LONG.(λ) Decimal degree	Ellip. Height (m)	M.S.L. Height (m)	N=h-H (m)	EGM08 (m)	Diff. (m)
GPS001	5.038400	7.002700	47.654	29.513	18.142	18.869	-0.727
GPS 02	4.988340	7.005440	42.542	24.294	18.248	18.904	-0.656
GPS 03	4.972250	6.951180	38.771	20.63	18.141	18.832	-0.691
GPS 04	4.988170	6.959680	41.357	23.096	18.261	18.833	-0.572
GPS 05	4.976870	6.950530	39.485	21.289	18.196	18.828	-0.632
GPS 06	4.968420	6.950770	38.351	20.218	18.133	18.834	-0.701
GPS 07	4.954950	6.947080	34.627	16.476	18.151	18.839	-0.688
GPS 08	4.953780	6.944280	36.819	18.648	18.171	18.836	-0.665
GPS 09	4.978020	6.968920	38.155	20.165	17.99	18.854	-0.864
GPS 10	4.976620	6.970370	39.661	21.445	18.216	18.857	-0.641
GPS 11	4.975170	6.971960	40.589	22.342	18.247	18.86	-0.613
GPS 12	4.953140	6.950450	35.359	17.181	18.178	18.845	-0.667
GPS 13	4.949710	6.952840	34.766	16.58	18.186	18.85	-0.664
GPS 14	4.946590	6.955110	34.756	16.568	18.188	18.856	-0.668
GPS 15	4.943010	6.957380	34.79	16.592	18.198	18.861	-0.663

GPS 16	4.939240	6.957960	34.784	16.569	18.215	18.865	-0.65
GPS 17	4.893160	6.964720	29.266	10.986	18.28	18.904	-0.624
GPS 18	4.894050	6.964340	29.87	11.58	18.29	18.903	-0.613
GPS 19	4.893300	6.966280	30.338	12.024	18.314	18.905	-0.591
GPS 20	4.875100	6.955990	32.335	14.017	18.318	18.906	-0.588
GPS 21	4.875640	6.954830	33.256	14.933	18.323	18.905	-0.582

The 3 D hybrid Geoid Model was developed using the Suffer version 13, and the result is as shown in Figure 2 to Figure 5. This satisfies objective two of this research.

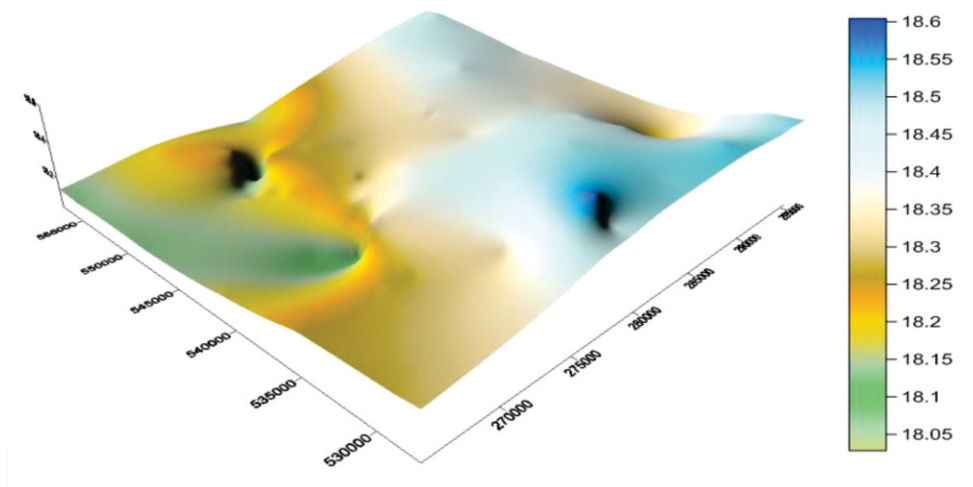


Figure 2: Kriging Hybrid Geoid Model

Figure 2 is the developed geoidal surface of part of Rivers State using the kriging interpolation model. Interestingly, the Kriging interpolation model is a smooth interpolator which is a critical requirement in the development of a geoidal surface. However, the Kriging interpolation model pays no homage to the observation points. That is, the values of the observation points are altered when generating the interpolation surface.

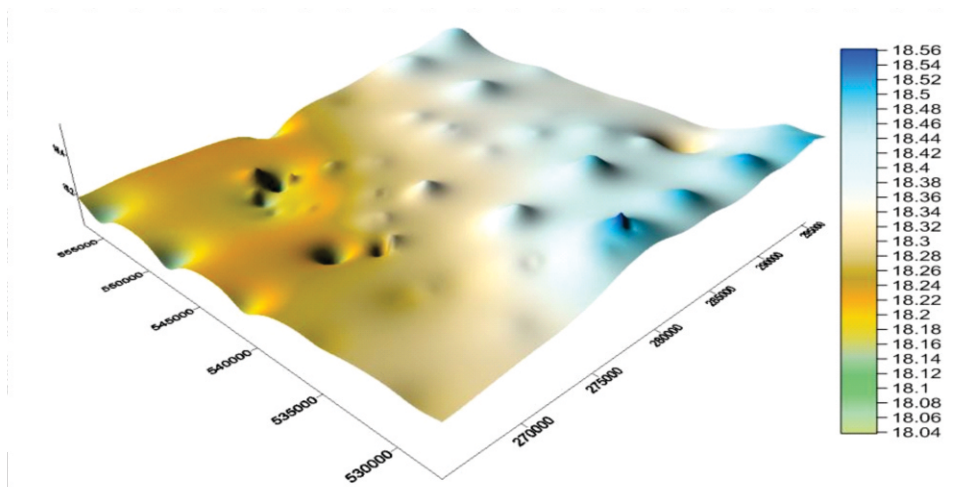


Figure 3: Inverse Distance Weighting Hybrid Geoid Model

Figure 3 present the hybrid geoid model of the study area developed using the IDW interpolation algorithms. The interesting part of the IDW is that it creates a surface that honors the observation points, that is the values of the observation point is not altered after the interpolation. This, however, results to the so-called bull eye effect, which are the sharp points on the surface model. As already established in physical geodesy, one of the fundamental qualities of a good geoid model is smoothness of the geoid surface, where all systematic effects have been filtered using gravity reduction techniques. The IDW however negate this quality.

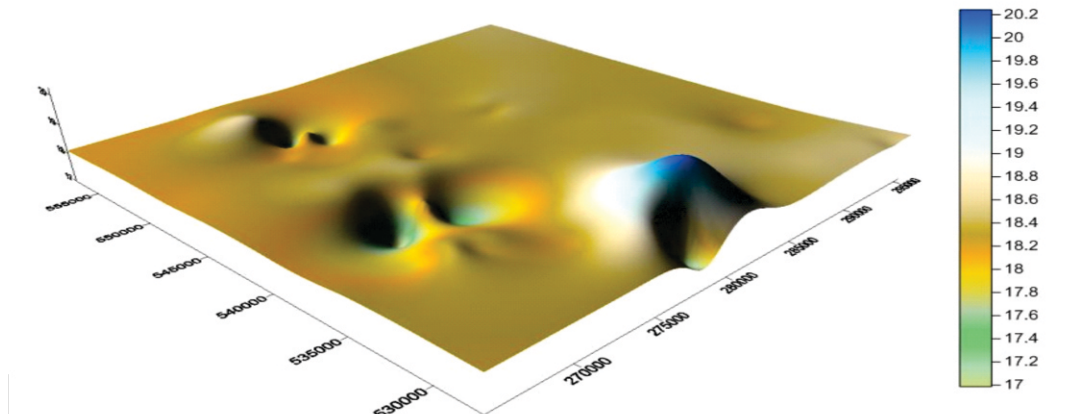


Figure 4: Radial Basis Function Hybrid Geoid Model

The radial basis Function hybrid geoid model is as shown in Figure 4, the weakness of this model for hybrid geoid modelling lies in the fact that it creates a surface that is over exaggerated on the minimal and maximal values of approximately 1m. This is indeed inadequate for precise geoid modelling where centimeter level accuracy is required. However, the RBF is a smooth interpolator.

The statistical investigation results of the three interpolation models is presented in Table 2 below:

Table 2: Statistical Result of the three Interpolation Models

Statics	Kriging	IDW	RBF
Mini. Value	18.028092	18.03857	16.98640471
Standard error	0.001132472	0.00082	0.002834069
Variance	0.012696669	0.006657	0.079516271
Max. Value	18.60439634	18.5616	20.24316444
RMSE	0.000645	0.00035	0.000894
MRE	0.007052253	0.005093	0.01365207
MAE	0.094285484	0.06875	0.158709067
Skwness	-0.158475583	-0.000157	1.871652031
Kurtosis	2.047691724	2.2465	14.75088901
Coeff. of Variation	0.006148126	0.0044538	0.015376964

From the statistical result in Table 2, the radial basis function gives a minimum and maximum value of 16.98640471 and 20.24316444, as against 18.028092, 18.03857 and 18.60439634, 18.5616 for Kriging and IDW which is about 1m deviation from the true value. It therefore implies that interpolation surface so generated using Radial Basis function though smooth as shown in Figure 4, is significantly exaggerated and hence not suitable for hybrid geoid determination.

All statistical investigation shows that the IDW gives a minimum deviation, and variance as shown in Table 2. However, as can be seen in Figure 3 one of the major disadvantages of the IDW is the bull eye effect which represent the concentric circle form around position of known spatial characteristic.

3.1 Validation of the Interpolation Models

In our quest to further validate the selected interpolation models, the IDW, and the Kriging, the degree of recoverability of the original values used for the interpolation was determined. For this purpose, five selected common point was used in the validation process. The validation involves determining the differences between a known point and an interpolated point. The result of the validation process is as shown in table 3.

Table 3: Shows the Validation of the Interpolation Models

SN	LAT. (ϕ) Decimal Degree	Long. (λ) Decimal Degree	Obs. (N=h-H)	Model Values (N) Using Kriging	Model values (N) Using IDW	Kriging Diff.	IDW Diff.
1	4.95585	7.15583	18.3851	18.4212	18.412	-0.0361	-0.0269
2	4.832328	6.944122	18.273	18.2815	18.282	-0.0085	-0.009
3	4.807219	6.97628	18.341	18.3731	18.373	-0.0321	-0.032
4	4.8933	6.96628	18.314	18.2896	18.289	0.0244	0.025
5	4.78799	7.15629	18.4876	18.4482	18.444	0.0394	0.0436
					RMS	0.00645	0.00035

The result of the validation process reveals that IDW gives a better recoverability than kriging, with a root mean square error of 0.00035 as against 0.00645 which is the RMS obtained using Kriging. It can also be seen that both methods do not give the exact values of the surface so models. Interpolation techniques at best are geostatistical approximation of the actual terrain characteristic. However, the accuracy of the selected interpolation algorithm increases proportionately with the common point i.e., the more information about the area we have, the better the result obtained from the interpolation.

The Kriging method of interpolation does not give the exact representation of the surface model. However, it has no bull's eye effect, and it also gives a significantly better statistical analysis than the radial basis function in that it uses the Lagrange minimization techniques to obtain the minimum variance estimates for the interpolated points. This makes it even more like the method of least squares collocation which is absolutely based on the principle of least squares.

4.0 Conclusion

This study investigates three selected interpolation techniques (Radial basis function, Inverse distance weighting, and Kriging interpolation models) used in geodesy and geosciences for the modelling of a spatial phenomenon such as geoid modelling. The result of the study shows that no interpolation techniques give a true representation of the model surface. The RBF is a smooth exaggerated interpolator, IDW honors the observation points and also gives minimal variance estimate of the interpolated points. However, it is affected by the bull eye effect which are concentric circle around the observation points making it not a smooth interpolator. The Kriging Interpolation on the other hand doesn't honors the observation point in the developed interpolation surface. However, it produces a smooth surface, an unbiased estimates and minimal variance of the interpolated points. Hence, the Kriging interpolation algorithms is best suitable for geoid modeling within the study area.

References

- Arana D., Camargo P. O., and Guimaraes G. N., (2017). Hybrid Geoid Model: Theory and Application in Brazil. *Annals of Brazilian Academy of Sciences* Vol. 89(3). ISSN: 1678-2690
- Basil D. D., (2021). The Development of a Hybrid Model for the Determination of the Component of the Deflection of the Vertical for Rivers State, Nigeria. An MSc Dissertation submitted to the Post Graduate School, Rivers State University.
- Davis J. C., (1986). *Statistics and Data Analysis in Geology*. John Wiley and Sons, New York.
- Deutsch C. V., and Journé J. L., (1992). *GSLIB- Geostatistical Software Library and Users Guide*, Oxford University Press, New York
- Fubara M. D. J., (2011). *Geodesy: The Backbone of the Science of Geoinformatics*. Fajemirokun F. A., (Ed) *Contemporary Issues in Surveying and Geoinformatics*. Bprint Publisher, 51, Remi-Fani Kayode Avenue, Off Oduduwa Street, GRA Ikeja, Lagos. ISBN:978-915-670-2.
- Hart L., Basil, D. D., and Oba, T., (2021). Assessment of Adjustments Methods in Traverse Networks for Positioning. *Nigerian Journal of Environmental Sciences and Technology*, ISSN:
- Hart L., (2015). Development of Datum Transformation Procedure for Nigeria Based on National Transformation Version 2 (NTv2) Model. An Unpublished Ph.D Thesis, submitted to the Department of Geoinformatics and Surveying, University of Nigeria, Enugu Campus, Nigeria.
- Herbert T. and Olatunji R. I., (2021). Determination of Orthometric Height using GNSS and EGM Data: A Scenario of the Federal University of Technology Akure. *International Journal of Environment and Geoinformatics*.

- Ikharo B. I, Ono, M.N., and Saheed, S., (2019). Establishment of Geodetic Network in Nigeria. *International Journal of Scientific Research in Science and Technology*, 6, 197-221.
- Kaplan E. D. and Hegarty C. J., (2017). Ed. *Understanding GPS/GNSS; Principles and Applications*. 3rd Edition. Artech house, Boston, London. ISBN-13:978-1-63081-0-580-0.
- Lee J., Kwon J., and Lee Y., (2021). Analyzing Precision and Efficiency of Global Navigation Satellite System-Derived Height Determination for Coastal and Island Areas. *Applied Sciences* Vol (11). Available online at <http://doi.org/10.3390/app11115310>.
- Moka E. C., (2011). Requirements for the Determination of Orthometric Heights from GPS-Determined Heights. As in Fajimirokun F. A. Ed., (2011). *Contemporary issues in Surveying and Geoinformatics*. Published by Bprint 51, Remi-FaniKayode Avenue, off Oduduwa Street, GRA Ikeja, Lagos. ISBN: 978-915-670-2.
- Nynke H., Malcolm H., Mark N., Phil J., and Christopher F., (2008). Comparison of six methods for the interpolation of daily, European climate data. *Journal of Geophysical Research*, Vol. 113, D21110, DOI: 10.1029/2008JD010100.
- Ogundere J. O., (2019). *Understand Least Squares Estimation and Geomatics Data Analysis*. John Wiley & Sons, Inc., 111 Rivers Street, Hoboken, NJ 07030, USA. ISBN: 9781119501442.
- Ojigi M. L., (2011). Determination of Suitable Terrain Surface Modeling Algorithms for Hydraulics Design of Storm Sewers. As in Fajimirokun F. A. Ed., (2011). *Contemporary issues in Surveying and Geoinformatics*. Published by Bprint 51, Remi-FaniKayode Avenue, off Oduduwa Street, GRA Ikeja, Lagos. ISBN: 978-915-670-2.
- Opaluwa Y. D. and Adejare Q. A., (2010). Derivation of Orthometric Heights from GPS measured Heights using Geometric Techniques and EGM96 Model. *Federal University of Technology Yola Journal of the Environment*. Vol. 5, No. 1.
- Pa'suya M. F., Md Din, A. H., Abbak, R. A. et al. (2022). Hybrid Geoid Model Over Peninsular Malaysia (PMHG2020) using two approaches. *Stud Geophysics Geoid* 66, 98-123. Available online at <https://doi.org/10.1007/S1200-021-0769-2>.
- Pavlis N, K., Holmes, S. A., Kenyon, S. C. and Factor, J. K., (2008). An Earth Gravitation Model to Degree 2160: EGM2008 presented at the 2008 General Assembly of the European Geosciences Union, Vienna, Austria.
- Tobler W. R., (1970). A Computer Movie Simulating Urban Growth in Detroit Region. *Economic Geography: Proceedings, International Geographical Union. Commission on Quantitative Method*. Stable URL: <http://www.jstor.org/stable/143141>. Accessed 15/07/2021 10:08am.

- Wang S., (2003). Artificial neural network. In: Interdisciplinary Computing in Java Programming, p. 81-100. Springer US.
- Willmott C.J. and Matsuura K, (2006). On the use of dimensioned measures of error to evaluate the performance of spatial interpolators. International Journal of Geographical Information Science, 20(1), 89-102.
- Willmott C.J. and Matsuura K, (2006). On the use of dimensioned measures of error to evaluate the performance of spatial interpolators. International Journal of Geographical Information Science, 20(1), 89-102.

Cite this article as:

Hart L., and Basil D. D. 2023. Comparison of Three Selected Interpolation Models for Hybrid Geoid Determination in Rivers State, Nigeria. *Nigerian Journal of Environmental Sciences and Technology*. 7(1) pp 191-203. <https://doi.org/10.36263/nijest.2023.01.0415>