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Displacement and Stress Analysis of Thick Rectangular Clamped Plates Using Third-Order Energy Functional

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ABSTRACT

This study analysed the stress and displacement responses of an all-clamped rectangular thick plate subjected to a uniformly distributed load using polynomial displacement functions. A polynomial shear deformation function was incorporated in the determination of the general governing equations; hence, there is no need for shear correction factors. Approximate Polynomial Displacement Functions w, u, and v for all the clamped thick rectangular plates subject to transverse loading were obtained. Numerical results were obtained using displacement equations obtained from the literature for the non-dimensional form of displacements and stresses of an all clamped (CCCC) plate at different aspect and span-depth ratios to determine the efficiency of this theory. The results obtained were validated as they showed good agreement with the results obtained by other researchers available in literature. Hence, this theory can be used as a reliable, concise, easy and dependable means for the stress and displacement analysis of thick plates. The results obtained also indicate that at a span-depth ratio of 100 and above, the classical plate theory (CPT) can be used to analyse plates, as the numerical values obtained are approximately equal to that of the CPT.

Keywords: Shear deformation, Displacement, Stress, CCCC, Thick plate

1.0. Introduction

A plate can be defined as a structural member confined by two parallel faces and a cylindrical surface called an edge (Onodagu *et al*., 2021). Plates are crucial components in engineering with a variety of applications ranging from aerospace engineering, civil and structural engineering, mechanical engineering, marine engineering and others. There is an increase in the use of plates in the modern engineering industry due to their advantages, including high strength and light weight. The analysis of engineering members is necessary to understand their behavior under loading conditions and hence, recommend optimum utilization of these components (Uzodinma *et al*., 2022).

The Kirchhoff (1850) classical plate theory (CPT) has widely been used to analyse thick and thin plates in structural mechanics. However, the major drawbacks of the CPT in the analysis of thick plates are the fact that the theory assumes the transverse shear deformation along the thickness of the plate to be zero and that the straight lines normal to the middle surface remain unchanged. Consequent to these shortcomings, several other theories have been proposed for the analysis of thick plates. Reissner (1945) and Mindlin (1951) were the pioneers of the widely known First-order shear deformation theories, where they assumed a constant transverse shear strain and hence the need for shear correction factors to describe the relationship between the shear strains and resultant shear forces (Sayyad, 2013). Other developed theories include Reddy's third order and other higher-order theories that take into consideration the transverse shear strain.

However, most of these theories are characterized by the use of trigonometric displacement functions, which, when in analysis, may result in complicated and erroneous mathematical calculations. Consequently, Ibearugbulam *et al*. (2012) developed polynomial displacement functions that provided a more concise, simplified and precise technique in the analysis of rectangular flat thin plates. Furthermore, Onyechere (2019) extended the use of these polynomial displacement functions to the vibration and stability analysis of thick rectangular plates, where he also developed a polynomial shear deformation function based on the Touratier (1991) trigonometric model. Onodagu *et al*. (2021) also used polynomial displacement functions for the flexural analysis of rectangular SSSS thick plates after developing a shear deformation function based on the Soldatos (1992) trigonometric model. This study aims to extend the Onadogu (2021) model to the flexural analysis of an all-clamped rectangular thick plate.

2.0. Theoretical framework

The polynomial shear deformation function developed by Onodagu *et al*. (2021) is expressed in Equation 1.

$$f(z) = z\left[1 - \frac{13}{10}\left(\frac{z}{t}\right)^2\right] \tag{1}$$

f (z) represents the shape function that describes the distribution of the transverse shear stresses along the thickness (z is the coordinate in the direction of the plate thickness) and “t” is the thickness of the plate.

If the dimensional coordinates x, y, and z are expressed in non-dimensional coordinates as;

$$x=iI \quad ; y=jJ \quad ; z=st \tag{2}$$

The expression for the deflection of rectangular thick plates as obtained by Onodagu *et al*. (2021) is expressed in Equation (3)

$$w(I,J) = w_x.w_y = (A_0 + A_1 I + A_2 I^2 + A_3 I^3 + A_4 I^4)(B_0 + B_1 J + B_2 J^2 + B_3 J^3 + B_4 J^4) \tag{3}$$

The coefficients A_m and B_n of the series are determined by the boundary conditions at the edges of the plate.

Similarly, the polynomial displacement functions for ‘u’ and ‘v’, for the thick plate as obtained by Onodagu *et al*. (2021) are expressed in Equations (4) and (5).

$$u = \frac{\partial w}{\partial I} \cdot St\left[-\frac{1}{i} + \left(1 - \frac{13S^2}{10}\right) \cdot \cap_1\right] \tag{4}$$

$$v = \frac{\partial w}{\partial J} \cdot St\left[-\frac{1}{j} + \left(1 - \frac{13S^2}{10}\right) \cdot \cap_2\right] \tag{5}$$

Where:

$$\cap_1 = \frac{200}{161i} \cdot \frac{1}{\int_0^1 \left(\frac{\partial \emptyset xx}{\partial I}\right) dJ} \cdot \left(\int_0^1 \left(\frac{\partial^2 w_y}{\partial J^2}\right) dQ\right) \frac{\partial w_x}{\partial I} \cdot \frac{200}{161i} \cdot \frac{1}{\left(\int_0^1 \left(\frac{\partial^2 \emptyset_{xy}}{\partial J^2}\right) dJ\right)} = constant \tag{6}$$

$$\cap_2 = n_3 \cdot \frac{200}{161ip} \cdot \frac{1}{\left(\int_0^1 \left(\frac{\partial \emptyset_{yy}}{\partial Q}\right) dJ\right)} \cdot \left(\int_0^1 \left(\frac{\partial^2 w_x}{\partial I^2}\right) dI\right) \frac{\partial^2 w_y}{p \partial J^2} n_3 \, \frac{200}{161i} \cdot \frac{1}{\left(\frac{\partial^2 \emptyset_{yx}}{\partial R^2}\right) dI} = constant \tag{7}$$

The Direct Governing Simultaneous Equations for the Flexural Analysis of isotropic thick rectangular plates subject to transverse loading where k_1, k_2 and k_3 are coefficients of displacements representing deflection, rotation in the x axis, and rotation in the y axis, respectively, are expressed in Equations (8), (9) and (10).

$$\frac{D}{i^4}\left[\left(m_1 + \frac{m_3}{P^4} + \frac{2m_2}{P^2}\right) K_1 + \left(-m_1 \frac{161}{200} - \frac{m_2}{P^2}\frac{161}{200}\right) K_2 + \left(-\frac{m_3}{P^4}\frac{161}{200} - \frac{m_2}{P^2}\frac{161}{200}\right) K_3\right] = qm_6 \tag{8}$$

$$\frac{D}{i^4}\left[\left(-m_1 \frac{161}{200} - \frac{m_2}{P^2}\frac{161}{200}\right) K_1 + \left(m_1 \frac{1223}{1867} + \left(\frac{1-\mu}{2P^2}\right) m_2 \frac{1223}{1867} + \left(\frac{1-\mu}{2}\right) \propto^2 m_4 \frac{12960}{2000}\right) K_2 + \left(\left(\frac{1+\mu}{2P^2}\right) m_2 \frac{1223}{1867}\right) K_3\right] \tag{9}$$

$$\frac{D}{i^4}\left[\left(-\frac{m_3}{P^4}\frac{161}{200} - \frac{m_2}{P^2}\frac{161}{200}\right) K_1 + \left(\left(\frac{1+\mu}{2P^2}\right) m_2 \frac{1223}{1867}\right) K_2 + \left(\frac{m_3}{P^4}\frac{1223}{1867} + \left(\frac{1-\mu}{2P^2}\right) m_2 \frac{1223}{1867} + \left(\frac{1-\mu}{2P^2}\right) \propto^2 m_5 \frac{12960}{2000}\right) K_3\right] \tag{10}$$

Where:

$$m_1 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial I^2}\right)^2 \partial I \, \partial J \tag{11}$$

$$m_2 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial I^2} \cdot \frac{\partial^2 h}{\partial J^2}\right) \partial I \, \partial J \tag{12}$$

$$m_3 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial J^2}\right)^2 \partial I \, \partial \mathrm{j} \tag{13}$$

$$m_4 = \int_0^1 \int^1 \left(\frac{\partial \mathrm{h}}{\partial \mathrm{I}}\right)^2 \partial I \, \partial J \tag{14}$$

$$m_5 = \int_0^1 \int_0^1 \left(\frac{\partial \mathrm{h}}{\partial \mathrm{J}}\right)^2 \partial I \, \partial J \tag{15}$$

$$m_6 = \int_0^1 \int_0^1 h \, \partial I \, \partial J \tag{16}$$

3.0. Results and Discussions

The approximate polynomial for the deflection "w" of an isotropic rectangular load for flexural analysis is presented as:

$$w(I,J) = w_x.w_y = (A_0 + A_1 I + A_2 I^2 + A_3 I^3 + A_4 I^4)(B_0 + B_1 J + B_2 J^2 + B_3 J^3 + B_4 J^4) \tag{17}$$

The boundary conditions for the CC edge are as follows;

i. At I = 0 and I =1 , the deflection and $w_x = 0$
ii. At I = 0 and I = 1, $Slope$ = 0, $i.e. \frac{\partial w_x}{\partial \mathrm{I}} = 0$

Substituting conditions

$i_0 = 0$; $i_1 = 0$; $i_3 = -2i_4$; $i_2 = i_4$ yields;

$$w_x = \mathrm{i}(I^2 - 2I^3 + I^4) \tag{18}$$

In the same manner, repeating the same procedure for simply clamped edge condition in the x-direction for the y-direction, we obtain a similar answer as expressed in Equation (19)

$$w_y = j_4(J^2 - 2J^3 + J^4) \tag{19}$$

The Polynomial Displacement Functions w, u and v for CCCC thick rectangular plates subject to transverse loading are expressed in Equations (20), (21) and (22) respectively.

$$w = i_4(I^2 - 2I^3 + I^4) \cdot j_4(\mathrm{J} - 2J^3 + J^4) \tag{20}$$

$$u = K_1[(2\mathrm{I} - 6I^2 + 4I^3)(J^2 - 2J^3 + J^4)].St\left[-\frac{1}{i} + \left(1 - \frac{13S^2}{10}\right) \cdot \cap_1\right] \tag{21}$$

$$v = K_1(I^2 - 2I^3 + I^4)(2\mathrm{J} - 6J^2 + 4J^3) \cdot St\left[-\frac{1}{j} + \left(1 - \frac{13S^2}{10}\right) \cdot \cap_2\right] \tag{22}$$

The shape function "h" of the CCCC thick plate is expressed in Equation (23).

$$h = (I - 2I^3 + I^4) \cdot (J - 2J^3 + J^4) \tag{23}$$

The "$m_{n=1,2\ldots6}$" values for the flexural analysis of the thick rectangular CCCC plate are summarized in Table 1.

Table 1: Summary of "$m_{n=1,2\ldots6}$" values for CCCC thick plate.

$m_{n=1,2\ldots6}$	value
m_1	0.00127
m_2	0.000363

m_3	0.00127
m_4	0.0000302
m_5	0.0000302
m_6	0.0011111

3.1. Numerical example

Determine the deflection, displacement, and stress of an isotropic CCCC plate with a varying span-to-depth ratio corresponding to different aspect ratios and subjected to a uniformly distributed load, q. μ = 0.3. Results are presented in the form;

$$\bar{\boldsymbol{w}} = \frac{100\mathrm{E}t^3}{\mathrm{q}i^4} w \qquad \overline{(\boldsymbol{u}, \boldsymbol{v})} = \frac{\mathrm{E}t^2}{\mathrm{q}i^3}(u, v) \qquad (\bar{\boldsymbol{\sigma}}_{\boldsymbol{x}}, \bar{\boldsymbol{\sigma}}_{\boldsymbol{y}}) = \frac{t^2}{\mathrm{q}i^2}(\sigma_x, \sigma_y)$$

$$\bar{\boldsymbol{\tau}}_{\boldsymbol{xy}} = \frac{t^2}{\mathrm{q}i^2}\tau_{xy} \qquad (\bar{\boldsymbol{\tau}}_{\boldsymbol{zx}}, \bar{\boldsymbol{\tau}}_{\boldsymbol{yz}}) = \frac{t}{\mathrm{q}i}(\tau_{zx}, \tau_{yz}) \tag{24}$$

The deflection, in-plane stresses and shear stress to be determined corresponds to;

$$\bar{w}\ at\ \left(x = \frac{i}{2}, y = \frac{j}{2}\right) \qquad \bar{u}\ at\ \left(x = 0, y = \frac{j}{2}\right) \qquad \bar{v}\ at\ at\ \left(x = \frac{i}{2}, y = 0\right)$$

$$(\bar{\sigma}_x, \bar{\sigma}_y)\ at\ \left(x = \frac{i}{2}, y = \frac{j}{2}\right) \qquad \bar{\tau}_{xy}\ at\ (x = 0.2, y = 0.2) \qquad \bar{\tau}_{zx}\ at\ \left(x = 0, y = \frac{j}{2}\right)$$

$$\bar{\tau}_{yz}\ at\ \left(x = \frac{i}{2}, y = 0\right) \tag{25}$$

The results obtained for the deflection at the center of the CCCC isotropic thick plate, the in-plane stresses, and the vertical shear stresses of the plate for different span-to-depth ratios corresponding to different aspect ratios (1, 1.5, 2,) subjected to uniformly distributed load are shown in Tables (2 – 4).

Table 2: Numerical results for the non-dimensional form of displacements and stresses for CCCC plates; p=1.

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}$	$\bar{\tau}_{yz}$
4	2.835239	-0.13388	-0.13388	2.868829	2.868829	-0.06074	1.694641	1.694641
5	2.339754	-0.12748	-0.12748	2.731658	2.731658	-0.05784	1.70094	1.70094
6	2.069055	-0.12398	-0.12398	2.656717	2.656717	-0.05625	1.704381	1.704381
7	1.905302	-0.12186	-0.12186	2.611384	2.611384	-0.05529	1.706463	1.706463
8	1.798806	-0.12049	-0.12049	2.581901	2.581901	-0.05467	1.707816	1.707816
9	1.725695	-0.11954	-0.11954	2.561661	2.561661	-0.05424	1.708746	1.708746
10	1.67335	-0.11887	-0.11887	2.54717	2.54717	-0.05393	1.709411	1.709411
15	1.549213	-0.11726	-0.11726	2.512804	2.512804	-0.0532	1.710989	1.710989
20	1.505711	-0.1167	-0.1167	2.50076	2.50076	-0.05295	1.711542	1.711542
25	1.485566	-0.11644	-0.11644	2.495183	2.495183	-0.05283	1.711798	1.711798
30	1.47462	-0.1163	-0.1163	2.492153	2.492153	-0.05277	1.711938	1.711938
40	1.463735	-0.11616	-0.11616	2.48914	2.48914	-0.0527	1.712076	1.712076
50	1.458697	-0.11609	-0.11609	2.487745	2.487745	-0.05267	1.71214	1.71214
100	1.451978	-0.11601	-0.11601	2.485885	2.485885	-0.05263	1.712225	1.712225
150	1.450733	-0.11599	-0.11599	2.48554	2.48554	-0.05263	1.712241	1.712241
200	1.450298	-0.11599	-0.11599	2.48542	2.48542	-0.05262	1.712247	1.712247
250	1.450096	-0.11598	-0.11598	2.485364	2.485364	-0.05262	1.712249	1.712249
300	1.449987	-0.11598	-0.11598	2.485334	2.485334	-0.05262	1.712251	1.712251
350	1.449921	-0.11598	-0.11598	2.485315	2.485315	-0.05262	1.712252	1.712252
400	1.449878	-0.11598	-0.11598	2.485304	2.485304	-0.05262	1.712252	1.712252
450	1.449848	-0.11598	-0.11598	2.485295	2.485295	-0.05262	1.712253	1.712253
500	1.449827	-0.11598	-0.11598	2.48529	2.48529	-0.05262	1.712253	1.712253
1000	1.44976	-0.11598	-0.11598	2.485271	2.485271	-0.05262	1.712254	1.712254

Table 3: Numerical results for the non-dimensional form of displacements and stresses for CCCC plates; p=1.5

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}$	$\bar{\tau}_{yz}$
4	4.398917	-0.21521	-0.18494	4.157126	3.096528	-0.11168	2.492575	1.661717
5	3.753186	-0.21099	-0.16983	4.037798	2.90961	-0.10565	2.543036	1.695357
6	3.396791	-0.20899	-0.16073	3.974835	2.799737	-0.1021	2.574319	1.716213
7	3.179688	-0.20791	-0.15488	3.937647	2.730106	-0.09987	2.594758	1.729839

8	3.037806	-0.20726	-0.15092	3.913863	2.683395	-0.09837	2.608731	1.739154
9	2.940062	-0.20684	-0.14813	3.897732	2.65063	-0.09733	2.618658	1.745772
10	2.869902	-0.20655	-0.14609	3.886285	2.626806	-0.09657	2.625939	1.750626
15	2.702882	-0.20592	-0.14112	3.859499	2.569015	-0.09473	2.643823	1.762549
20	2.64413	-0.20572	-0.13932	3.850238	2.548314	-0.09408	2.650305	1.76687
25	2.616882	-0.20563	-0.13849	3.845972	2.538645	-0.09377	2.653346	1.768898
30	2.602066	-0.20558	-0.13803	3.843661	2.533369	-0.0936	2.655009	1.770006
40	2.587325	-0.20553	-0.13757	3.841366	2.528107	-0.09344	2.65667	1.771114
50	2.580498	-0.20551	-0.13736	3.840306	2.525666	-0.09336	2.657442	1.771628
100	2.571392	-0.20548	-0.13707	3.838893	2.522405	-0.09326	2.658473	1.772315
150	2.569706	-0.20547	-0.13702	3.838631	2.5218	-0.09324	2.658664	1.772443
200	2.569115	-0.20547	-0.137	3.83854	2.521589	-0.09323	2.658731	1.772488
250	2.568842	-0.20547	-0.13699	3.838498	2.521491	-0.09323	2.658762	1.772508
300	2.568693	-0.20547	-0.13699	3.838475	2.521438	-0.09323	2.658779	1.77252
350	2.568604	-0.20547	-0.13699	3.838461	2.521405	-0.09323	2.658789	1.772526
400	2.568546	-0.20547	-0.13698	3.838452	2.521385	-0.09323	2.658796	1.772531
450	2.568506	-0.20547	-0.13698	3.838446	2.52137	-0.09322	2.658801	1.772534
500	2.568478	-0.20547	-0.13698	3.838441	2.52136	-0.09322	2.658804	1.772536
1000	2.568386	-0.20547	-0.13698	3.838427	2.521328	-0.09322	2.658814	1.772543

Table 4: Numerical results for the non-dimensional form of displacements and stresses for CCCC plates; p=2.0

α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}$	$\bar{\tau}_{yz}$
4	5.298245	-0.26408	-0.18534	4.811168	2.833413	-0.144	2.913429	1.456714
5	4.520372	-0.25786	-0.16498	4.658385	2.634864	-0.13335	2.956355	1.478178
6	4.090694	-0.25458	-0.15312	4.575004	2.520888	-0.12722	2.981456	1.490728
7	3.828971	-0.25264	-0.14566	4.524592	2.449863	-0.1234	2.997262	1.498631
8	3.657984	-0.2514	-0.1407	4.491818	2.402783	-0.12087	3.007807	1.503904
9	3.540229	-0.25055	-0.13723	4.469322	2.37004	-0.1191	3.015173	1.507586
10	3.455732	-0.24995	-0.13472	4.453218	2.346381	-0.11783	3.020511	1.510255
15	3.254689	-0.24854	-0.12867	4.415033	2.289533	-0.11476	3.033391	1.516696
20	3.184013	-0.24806	-0.12651	4.401654	2.269358	-0.11367	3.037981	1.51899
25	3.151244	-0.24783	-0.12551	4.395459	2.25997	-0.11317	3.04012	1.52006
30	3.133429	-0.24771	-0.12496	4.392093	2.254858	-0.11289	3.041285	1.520643
40	3.115704	-0.24759	-0.12442	4.388745	2.249764	-0.11262	3.042447	1.521224
50	3.107497	-0.24753	-0.12417	4.387196	2.247404	-0.11249	3.042986	1.521493
100	3.09655	-0.24746	-0.12383	4.38513	2.244253	-0.11232	3.043705	1.521852
150	3.094522	-0.24744	-0.12377	4.384747	2.24367	-0.11229	3.043838	1.521919
200	3.093813	-0.24744	-0.12374	4.384613	2.243465	-0.11228	3.043885	1.521942
250	3.093484	-0.24744	-0.12373	4.384551	2.243371	-0.11227	3.043906	1.521953
300	3.093306	-0.24743	-0.12373	4.384518	2.243319	-0.11227	3.043918	1.521959
350	3.093198	-0.24743	-0.12373	4.384497	2.243288	-0.11227	3.043925	1.521963
400	3.093128	-0.24743	-0.12372	4.384484	2.243268	-0.11227	3.04393	1.521965
450	3.093081	-0.24743	-0.12372	4.384475	2.243254	-0.11227	3.043933	1.521967
500	3.093046	-0.24743	-0.12372	4.384469	2.243244	-0.11226	3.043935	1.521968
1000	3.092937	-0.24743	-0.12372	4.384448	2.243213	-0.11226	3.043942	1.521971

From the numerical results for the non-dimensional form of displacements and stresses for all the shape functions considered, it is observed that as the span-depth ratio increases, values of in-plane quantities and those of out-of-plane quantities decrease for each aspect ratio, p. However, this decrease was sharp until α=100, where values obtained were approximately not different. This is further buttressed graphically in Figures 1, 2 and 3. Similarly, it can also be observed that as the aspect ratio increases, the values of in-plane quantities and those of out-of-plane quantities increase also.

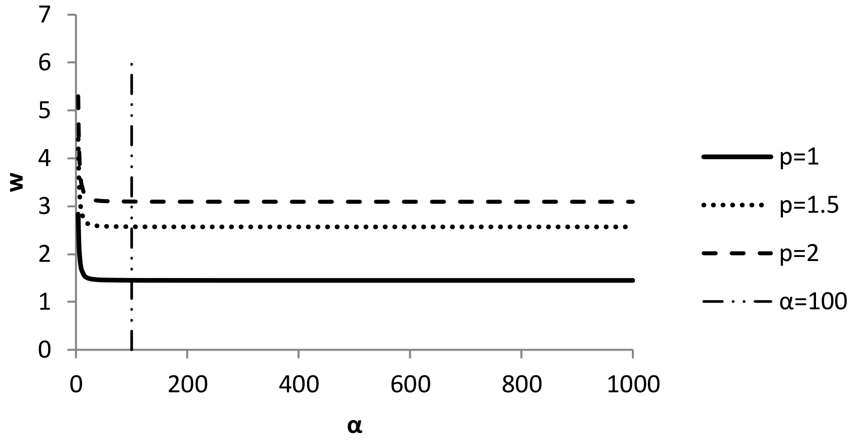

Figure 1: A graph of non-dimensional deflection, $\overline{w}$ against span-depth ratio.

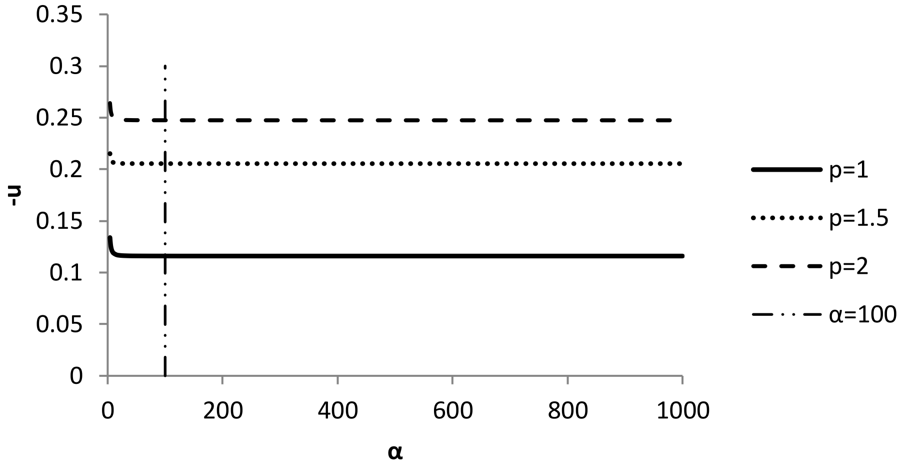

Figure 2: A graph of non-dimensional displacement, $-\bar{u}$ against span-depth ratio.

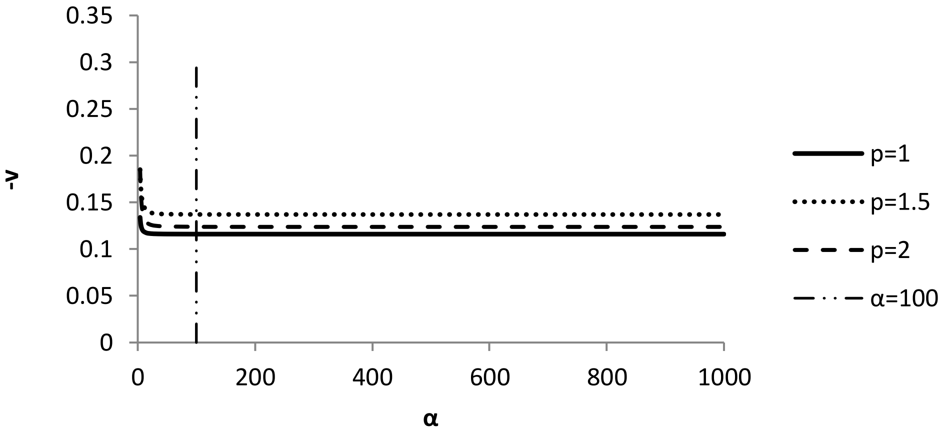

Figure 3: A graph of non-dimensional displacement, $-\bar{v}$ against span-depth ratio.

Table 5: Non dimensional forms of displacement of CCCC plate for α = 5 and $p = \frac{j}{i} = 1$

Study	α	$\bar{w} = w\left(\frac{D}{qi^4}\right)x100$	$\bar{u} = u\left(\frac{D}{qi^4}\right)x100$	$\bar{v} = v\left(\frac{D}{qi^4}\right)x100$
present	5	0.2143	-0.233	-0.233
Ibearugbulem *et al.* (2018)		0.2144	-0.225	-0.225
% difference		0.04664179	-3.55556	-3.55556
Zhong and Xu (2017)		0.2114	-	-
% difference		-1.371807001	-	-
Lok and Cheng (2001)		0.2147	-	-
% difference		0.186306474	-	-
Liu and Liew (1998)		0.2172	-	-
% difference		1.335174954	-	-
Shen and He (1995)		0.2204	-	-
% difference		2.7676951	-	-
Rui *et al.* (2014)		0.2172	-	-
% difference		1.335174954	-	-

Table 6: Non dimensional forms of displacement of CCCC plate for α = 10 and $p = \frac{j}{i} = 1$

Study	α	$\bar{w} = w\left(\frac{D}{qi^4}\right)x100$	$\bar{u} = u\left(\frac{D}{qi^4}\right)x100$	$\bar{v} = v\left(\frac{D}{qi^4}\right)x100$
present	10	0.1532	-0.109	-0.109
Ibearugbulem *et al.* (2018)		0.1534	-0.2094	-0.2094
% difference		0.1303781	47.94651	47.94651
Zhong and Xu (2017)		0.1483	-	-
% difference		-3.304113284	-	-
Lok and Cheng (2001)		0.1495	-	-
% difference		-2.474916388	-	-
Liu and Liew (1998)		0.1505	-	-
% difference		-1.794019934	-	-
Shen and He (1995)		0.1513	-	-
% difference		-1.255783212	-	-
Rui *et al.* (2014)		0.1505	-	-
% difference		-1.794019934	-	-

Table 5 and 6 presents a comparison between numerical results obtained for the non- dimensional deflection and in-plane displacements of the CCCC plate shape function considered in this study and those of other researchers available in literature. Results obtained herein showed good agreement with those of the other researchers for α = 5, with a maximum percentage difference of 2.7676951% (Shen and He, 1995) for non-dimensional deflection, $\bar{w}$ and -3.55556% (Ibearugbulem *et al.*, 2018) for non-dimensional displacements, $\bar{u}$ *and* $\bar{v}$. This represents a 97% and 96% level of agreement for the results of the non- dimensional deflection and in-plane displacements obtained in this study with those of the other researchers considered. A similar occurrence was observed for the non-dimensional deflection, $\bar{w}$ at α = 10, where a maximum percentage difference of -3.304113284 (Zhong and Xu, 2017) was obtained, indicating 96.5% agreement. However, this was not so for the in-plane displacements at α = 10, where a significant percentage difference of 47.94651% (Ibearugbulem *et al*., 2018) was observed.

4.0. Conclusions

This study analyzed the stress and displacement responses of an all clamped rectangular thick plate subjected to a uniformly distributed load using polynomial displacement functions. The mathematical technique employs a simple but precise method that involves definite integration of obtained solutions in the form of differential equations. These differential equations are satisfied throughout the domain of the plate.

The equations and constants obtained in this study have been validated as they show good agreement with other research results available in the literature. To this effect, they can be used as a reliable and dependable means for the flexural analysis of thick plates.

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